

Dimuonium ($\mu^+\mu^-$) Production in a Quark-Gluon Plasma

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We study dimuonium ($\mu^+\mu^-$) production in the quark-gluon plasma created in relativistic heavy ion collisions. The production is controlled by the process $q\bar{q} \rightarrow (\mu^+\mu^-)g$, and the dimuonium motion in the plasma is described by a transport equation. While the electrodynamics dominated dimuonium yield is not high enough, the transverse energy distribution carries the information of the plasma at RHIC and LHC energies.

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The observation of positronium (e^+e^-) [1] and the clarification of the leptonic nature of muons lead to a natural question that whether there are “true muonium” [2] or “dimuonium” [3, 4] states ($\mu^+\mu^-$). Note that the terminology “muonium” has been signed to the bound state (μ^+e^-), which has been discovered in 1960 [5]. Since dimuonium is an ideal system of quantum electrodynamics (QED), many of its properties are theoretically predicted [2, 6] with standard calculations of QED. For instance, dimuonium is one of the most compact QED systems with Bohr radius 512 fm, the binding energy of the ground state ($1s$) is 1.4 KeV, and the life times for the states 1S_0 and 3S_1 are respectively 0.602 ps (decay to $\gamma\gamma$) and 1.81 ps (decay to e^+e^-), which are both much shorter than the life time of muon (2.197 μ s) and thus the muon weak decay can be ignored in the production of dimuonium. Many dimuonium production mechanisms have been proposed, like direct $\mu^+\mu^-$ collision [2], $e^+e^- \rightarrow (\mu^+\mu^-)$ [7], $\pi^-p \rightarrow (\mu^+\mu^-)n$ [6], $\gamma A \rightarrow (\mu^+\mu^-)A$ [6] and $eA \rightarrow e(\mu^+\mu^-)A$ [8], where A stands for a nucleus. Dimuoniums can also be produced in heavy ion collisions through a pure electromagnetic process $A_1A_2 \rightarrow A_1A_2(\mu^+\mu^-)$, where the mechanism is the fusion of coherent photons emitted from the nuclei [9]. Recently, the possibility of dimuonium production at modern electron-positron colliders is investigated [10]. However, dimuonium has not yet been experimentally discovered.

It is generally believed that there is a quantum chromodynamics (QCD) phase transition in hot and dense nuclear matter, which is related to the deconfinement process in moving from a hadron gas to a quark-gluon plasma (QGP) [11]. The realization of such a phase transition in laboratories can only be through relativistic heavy ion collisions [12]. Considering the decreasing temperature and density during the rapid expansion of the fireball formed in the collisions, the QGP can not be measured directly in the cold and dilute final state, and one needs signatures to identify the QGP formation. Since leptons interact with particles only through electromagnetic channel, the leptons produced in the QGP carry the information of the plasma and will not lose it when they pass through the collision region to reach the detectors. Taking into account the fact that the lepton number in the fireball increases with the temperature of

the plasma, the probability to produce a dimuonium state in high energy nuclear collisions should be much larger than that in elementary electron-positron and nucleon-nucleon collisions. In this paper we study the dimuonium production in a hot QGP and calculate the production rate in high energy nuclear collisions at relativistic heavy ion collider (RHIC) and large hadron collider (LHC) [12]. Since the dimuonium life time is much longer than the QGP life time (about several fm/c), the dimuonium decay and dissociation in the QGP can safely be neglected. In our calculation we will also ignore the dimuonium production in the initial state and in the hadron gas, since their contribution should be much smaller in comparison with the production in the QGP.

There are two kinds of production processes in QGP for the dimuonium states ($\mu^+\mu^-$), one is from the lepton pair annihilation and the other is from the quark-antiquark pair annihilation. While the QCD processes in vacuum at RHIC and LHC energies may still be non-perturbative, the medium effects will largely reduce the effective coupling constant α_s (~ 0.3) [13] and may make the perturbative analysis available. By counting the number of vertexes and considering the phase space suppression, one can simply estimate the order of the dimuonium production cross sections in different processes. The cross sections of the two processes at leading order, $l^+l^- \rightarrow (\mu^+\mu^-)$ and $q\bar{q} \rightarrow (\mu^+\mu^-)$, are of the order of α^2 , with α being the electromagnetic coupling constant. Due to the requirement of energy conservation, only the initial states in a narrow energy window around the binding energy 1.4 KeV contribute to the production processes, and therefore the both cross sections at leading order become very small. The next leading process is $q\bar{q} \rightarrow (\mu^+\mu^-)g$ with the cross section of the order of $\alpha_s\alpha^2$. The subsequent processes are $l^+l^-(q\bar{q}) \rightarrow (\mu^+\mu^-)\gamma$ and $q\bar{q} \rightarrow (\mu^+\mu^-)gg$. The former is of the order of α^3 and the latter is of the order of $\alpha_s^2\alpha^2$. Since the latter includes three particles in the final state, the process is strongly suppressed in comparison with the former. Other processes are higher order contributions. Therefore, we consider in the following the main process $q\bar{q} \rightarrow (\mu^+\mu^-)g$ for the dimuonium production in the QGP. The contribution from the other processes is at least ten times smaller from the simple vertex counting.

The Feynman diagrams at tree level for the main pro-

duction process $q\bar{q} \rightarrow (\mu^+\mu^-)g$ are shown in Fig.1. Since strange quarks are much heavier than light quarks, the amount of strange quarks in QGP is less than 10% of the amount of light quarks at RHIC and LHC energies, and we consider only light quarks in the calculation ($N_f = 2$), and their current mass is set to be zero. The scattering amplitude $\overline{\mathcal{M}}(q_1q_2 \rightarrow p_1p_2k)$ for the process $q\bar{q} \rightarrow \mu^+\mu^-g$ can be written as

$$\begin{aligned} \overline{\mathcal{M}} = & gQe^2\bar{v}(q_2)\left[t\gamma^\mu\frac{1}{\not{q}_2-\not{k}}\gamma^\nu+\gamma^\nu\frac{1}{\not{q}_1-\not{k}}t\gamma^\mu\right] \\ & \times u(q_1)\frac{g_{\nu\rho}}{(p_1+p_2)^2}\bar{u}(p_1)\gamma^\rho v(p_2)\epsilon_\mu^*(k), \end{aligned} \quad (1)$$

where q_1, q_2, p_1, p_2 and k are respectively the momenta of q, \bar{q}, μ^+, μ^- and g , Q is the quark charge number, t is the color matrix, g is related to the effective coupling constant via the definition $\alpha_s = g^2/(4\pi)$, u, v, \bar{u} and \bar{v} are Dirac spinors, and ϵ_μ is the gluon polarization vector satisfying $\epsilon_\mu^*(k)\epsilon_\nu(k) = -g_{\mu\nu} + k_\mu k_\nu/m_g^2$ with thermal gluon mass $m_g = 2gT/3$ in the deconfinement phase [14]. For simplicity, we have dropped in (1) the flavor, color and spin indices of quarks and gluons.

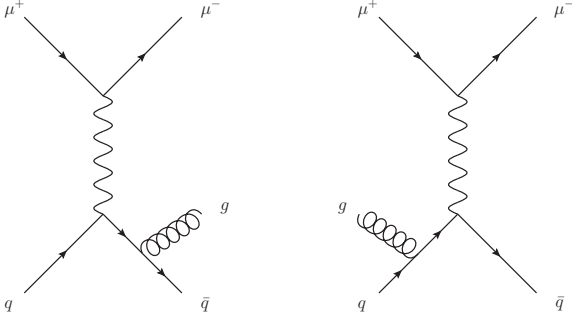


FIG. 1: The Feynman diagrams at tree level for the main dimuonium production process $q\bar{q} \rightarrow (\mu^+\mu^-)g$ in QGP.

Introducing the total and relative momenta $p = p_1 + p_2$ and $p' = (p_1 - p_2)/2$ of μ^+ and μ^- , the amplitude $\overline{\mathcal{M}}(q_1q_2 \rightarrow p_1p_2k)$ can be expressed as $\overline{\mathcal{M}}(q_1q_2 \rightarrow pp'k)$. Since the bound state $(\mu^+\mu^-)$ is non-relativistic, we can compute its production amplitude $\mathcal{M}(q_1q_2 \rightarrow pk)$ by integrating out the relative momentum p' in the center-of-mass frame of the dimuonium [15],

$$\begin{aligned} \mathcal{M} = & \sqrt{\frac{2}{m_d}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \overline{\mathcal{M}}(q_1q_2 \rightarrow pp'k) \psi^*(\mathbf{p}'), \\ \psi(\mathbf{p}') = & \frac{8\sqrt{\pi}a_0^{3/2}}{(1+a_0^2\mathbf{p}'^2)^2}, \end{aligned} \quad (2)$$

where m_d is the dimuonium mass, and $\psi(\mathbf{p}')$ is the relative wave function for the ground state in momentum space with a_0 being the Bohr radius. From the known scattering amplitude, we can calculate the dimuonium production cross section $\sigma_{q\bar{q}}^{(\mu^+\mu^-)g}(s)$ as a function of

$s = (q_1 + q_2)^2$ and the transition probability

$$W_{q\bar{q}}^{(\mu^+\mu^-)g}(s) = \frac{16\pi s^2}{\sqrt{(s - m_d^2 - m_g^2)^2 - 4m_d^2m_g^2}} \sigma_{q\bar{q}}^{(\mu^+\mu^-)g}(s). \quad (3)$$

The medium created in high-energy nuclear collisions evolves dynamically. In order to extract information about the medium by analyzing the dimuonium distributions, both the hot and dense medium and the dimuonium production process must be treated dynamically. In this paper, we treat continuous dimuonium production in QGP self-consistently, including hydrodynamic evolution of the QGP. Since dimuonium is a pure electromagnetic system, it can not be thermalized with the medium which is governed by strong interactions. Thus its phase space distribution should be controlled by a transport equation. The transport equation should then be solved together with the hydrodynamic equation which characterizes the space-time evolution of the QGP. Considering that dimuonium is a heavy bound state, we use a classical Boltzmann-type transport equation to describe its evolution. The distribution $f(\mathbf{p}_t, y, \mathbf{x}_t, \eta, \tau|\mathbf{b})$ as a function of transverse momentum \mathbf{p}_t and coordinate \mathbf{x}_t , longitudinal rapidity y and space-time rapidity η and proper time τ at fixed impact parameter \mathbf{b} in a heavy ion collision is characterized by the equation

$$\left[\cosh(y - \eta) \frac{\partial}{\partial \tau} + \frac{1}{\tau} \sinh(y - \eta) \frac{\partial}{\partial \eta} + \mathbf{v}_t \cdot \nabla_t \right] f = \beta \quad (4)$$

with the dimuonium transverse velocity $\mathbf{v}_t = \mathbf{p}_t/E_t$ and transverse energy $E_t = \sqrt{m_d^2 + \mathbf{p}_t^2}$. Because the life time of dimuonium is much longer than the life time of QGP, the decay of dimuonium is ignored. Moreover, since dimuonium does not participate in strong interactions, the generated dimuonium interacts with QGP only electromagnetically, the dissociation can then be neglected too. Therefore, we consider only the gain term $\beta(\mathbf{p}_t, y, \mathbf{x}_t, \eta, \tau|\mathbf{b})$ on the right hand side of the transport equation,

$$\begin{aligned} \beta = & \frac{1}{2E_t} \int \frac{d^3\mathbf{k}}{(2\pi)^3 2E_g} \frac{d^3\mathbf{q}_1}{(2\pi)^3 2E_q} \frac{d^3\mathbf{q}_2}{(2\pi)^3 2E_{\bar{q}}} W_{q\bar{q}}^{(\mu^+\mu^-)g}(s) \\ & \times f_q f_{\bar{q}} (1 + f_g) (2\pi)^4 \delta^{(4)}(p + k - q_1 - q_2), \end{aligned} \quad (5)$$

where $E_q = \sqrt{m_q^2 + \mathbf{q}_1^2}$, $E_{\bar{q}} = \sqrt{m_q^2 + \mathbf{q}_2^2}$ and $E_g = \sqrt{m_g^2 + \mathbf{k}^2}$ are respectively quark, anti-quark and gluon energies, and $f_q, f_{\bar{q}}$ and f_g are the thermal distributions for quarks and gluons, $f_q = 1/(e^{q_1^\mu u_\mu/T} + 1)$, $f_{\bar{q}} = 1/(e^{q_2^\mu u_\mu/T} + 1)$ and $f_g = 1/(e^{k^\mu u_\mu/T} - 1)$. By using Bjorken's hydrodynamics [16], the fluid velocity u_μ appeared in the distributions and the temperature T in the distributions and gluon mass are functions of coordinates (\mathbf{x}_t, η) at fixed \mathbf{b} and determined by the ideal hydrodynamic equations [17]

$$\partial_\tau E + \nabla \cdot \mathbf{M} = -(E + p)/\tau,$$

$$\begin{aligned}
\partial_\tau M_x + \nabla \cdot (M_x \mathbf{v}) &= -M_x/\tau - \partial_x p, \\
\partial_\tau M_y + \nabla \cdot (M_y \mathbf{v}) &= -M_y/\tau - \partial_y p, \\
\partial_\tau R + \nabla \cdot (R \mathbf{v}) &= -R/\tau
\end{aligned} \tag{6}$$

with the Lorentz factor $\gamma = 1/\sqrt{1-\mathbf{v}^2}$ and definitions $E = (\epsilon+p)\gamma^2 - p$, $\mathbf{M} = (\epsilon+p)\gamma^2 \mathbf{v}$ and $R = \gamma n$ as functions of energy density ϵ , pressure p and baryon density n of the medium. To close the hydrodynamic equations, we need an equation of state to describe the nature of the QGP. We take the result from the lattice QCD simulation with a phase transition temperature of deconfinement $T_c = 190$ MeV [18]. Since the colliding energy is so high in heavy ion collisions at RHIC and LHC, the net baryon density in the fireball is rather small [12], we can simply set $n = 0$ in the numerical calculations. The initial condition of the hydrodynamic equations is controlled by the nuclear geometry which determines the impact parameter and the colliding energy which governs the ratio of soft to hard contributions [19].

Since we did not consider the initial dimuonium production before the QGP formation and have neglected the loss term in the transport equation (4), its analytic solution becomes simple,

$$\begin{aligned}
f(\mathbf{p}_t, y, \mathbf{x}_t, \eta, \tau | \mathbf{b}) &= \int_{\tau_0}^{\tau} d\tau' \frac{\beta(\mathbf{p}_t, y, \mathbf{X}_t(\tau'), H(\tau'), \tau' | \mathbf{b})}{\Delta(\tau')} \\
&\times \Theta(T(\mathbf{X}_t, H, \tau' | \mathbf{b}) - T_c)
\end{aligned} \tag{7}$$

with the definitions

$$\begin{aligned}
\mathbf{X}_t(\tau') &= \mathbf{x}_t - \mathbf{v}_t [\tau \cosh(y - \eta) - \tau' \Delta(\tau')], \\
H(\tau') &= y - \text{arcsinh}(\tau/\tau' \sinh(y - \eta)), \\
\Delta(\tau') &= \sqrt{1 + (\tau/\tau')^2 \sinh^2(y - \eta)},
\end{aligned} \tag{8}$$

where the local temperature $T(\mathbf{X}_t, H, \tau' | \mathbf{b})$ as a function of time and coordinates at fixed impact parameter is determined by the evolution of the medium (6), the step function Θ indicates that the mechanism of generating dimuonium discussed here can only take place in deconfined region, and the coordinate shifts $\mathbf{x}_t \rightarrow \mathbf{X}_t$ and $\eta \rightarrow H$ in the solution (7) reflect the leakage effect in the transverse and longitudinal directions. The time integration is from the initial time τ_0 to τ . By integrating the distribution over the phase space, we obtain the dimuonium transverse momentum distribution at fixed impact parameter \mathbf{b} . For the finally observed dimuonium distribution, τ should be so chosen that it is not earlier than the end time τ_f of the QGP.

$$\begin{aligned}
f(p_t | \mathbf{b}) &= \frac{dN(\mathbf{b})}{2\pi p_t dp_t} \\
&= \frac{\tau_c}{(2\pi)^3} \int d^2 \mathbf{x}_t dy d\eta E_t \cosh(y - \eta) \\
&\times f(\mathbf{p}_t, y, \mathbf{x}_t, \eta, \tau_c | \mathbf{b}),
\end{aligned} \tag{9}$$

where τ_c is an arbitrary time after τ_f , since the momentum distribution is unchanged for all $\tau > \tau_f$.

We can define the transverse energy distribution $f(E_t | \mathbf{b}) = dN(\mathbf{b})/(2\pi E_t dE_t)$. It is shown in Fig. 2 for central ($b=0$) Au+Au collisions at RHIC energy $\sqrt{s_{NN}} = 200$ GeV and Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 5.5$ TeV. We have taken the effective coupling constant $\alpha_s = 0.3$ (corresponding to $T/T_c \simeq 1.5 - 2$ [13]) and the dimuonium mass $m_d \simeq 2m_\mu = 211$ MeV (neglecting the binding energy 1.4 KeV) in the numerical calculations. While the dimuoniums distribute in a wider region at LHC, they behave similarly at two energies. The result in the low E_t region of $E_t < 1$ GeV can be parameterized as a thermal distribution $f(E_t) \sim e^{-E_t/T_{eff}}$ with a slope parameter $T_{eff} = 195$ MeV at RHIC and 240 MeV at LHC. This indicates that the thermodynamic information of the medium carried by quarks and gluons is partly inherited by the produced dimuoniums and can be used to signal the QGP formation in high energy nuclear collisions.

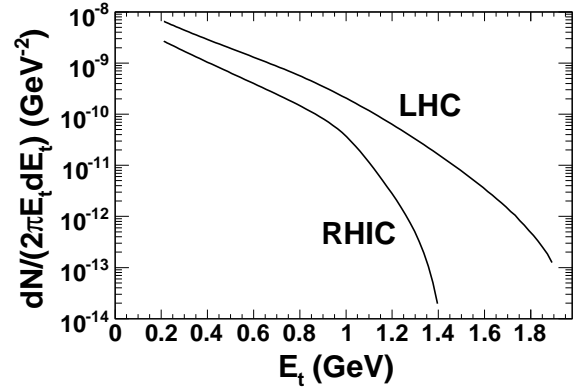


FIG. 2: The dimuonium transverse energy distribution $dN/(2\pi E_t dE_t)$ in central Au+Au collisions at RHIC energy $\sqrt{s_{NN}} = 200$ GeV and Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 5.5$ TeV.

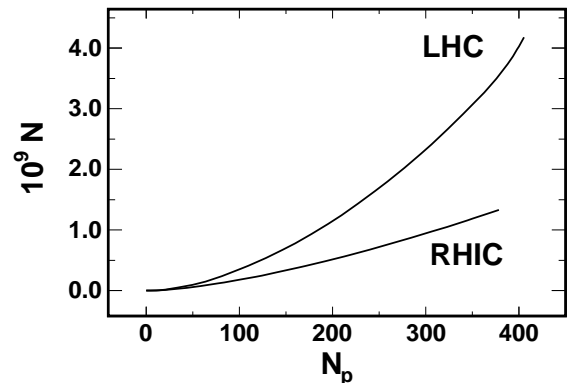


FIG. 3: The rescaled dimuonium number $10^9 N$ as a function of participant number N_p in Au+Au collisions at RHIC energy $\sqrt{s_{NN}} = 200$ GeV and Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 5.5$ TeV.

Fig.3 shows the momentum integrated dimuonium yield $N(\mathbf{b}) = 2\pi \int f(p_t|\mathbf{b})p_t dp_t$ as a function of the number of participant nucleons N_p in heavy ion collisions at RHIC and LHC energies. The relation between N_p and the impact parameter \mathbf{b} can be easily determined by the nuclear geometry. With increasing centrality, the participant number increases, and the temperature, the life time and the space region of the formed QGP increase. As a result, the dimuonium yield goes up with centrality, due to the enhancement of the quark and gluon numbers. For central collisions with maximum N_p , the dimuonium number is 1.3×10^{-9} at RHIC energy and becomes about 3 times larger at LHC energy, as shown in Fig.3.

Let's now compare the two dimuonium production mechanisms in heavy ion collisions. One is through the pure electromagnetic channel, $A_1 A_2 \rightarrow A_1 A_2 (\mu^+ \mu^-)$, proposed by Ginzburg et al. [9], and the other is inside the formed QGP, $q\bar{q} \rightarrow (\mu^+ \mu^-)g$, discussed here. For impact parameter $b > 2R_A$, where R_A is the colliding nuclear radius, there is no QGP formed, the production is only through the electromagnetic channel. However, for $b < 2R_A$, the colliding nuclei are broken, and the assumption of the replacement of the perturbation parameter α by $Z\alpha$ with each photon exchange is no longer

valid for the electromagnetic channel. For $b \ll 2R_A$, the QGP is formed with a high temperature, long life time and large size, the process $q\bar{q} \rightarrow (\mu^+ \mu^-)g$ becomes dominant. Since the pure electromagnetic channel is not related to the fireball, the transverse energy distribution in this channel can not show the thermodynamic behavior.

In summary, we investigated the dimuonium production in the QGP formed in relativistic heavy ion collisions. The dimuonium motion in the QGP is described by a transport equation with the gain term characterized by the production process $q\bar{q} \rightarrow (\mu^+ \mu^-)g$, and the space-time evolution of the plasma is controlled by ideal hydrodynamic equations. By solving the coupled transport and hydrodynamic equations for high energy nuclear collisions at RHIC and LHC energies, we found that while the electrodynamics dominated dimuonium yield is not high enough, the transverse energy distribution inherits the thermodynamic behavior of the hot medium and can be considered as an electromagnetic probe of the QGP.

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